Comparison of Theory and Simulation for a Radially-Symmetric Transit-Time Oscillator

J.W. Luginsland
Air Force Research Laboratory, Kirtland AFB New Mexico 87117-5776
E. Schamiloglu, R.L. Wright
University of New Mexico, Albuquerque, New Mexico 87131

Abstract

The transit-time effect in a coaxial structure has been used by Arman [1] to design low impedance high power microwave devices that use no externally generated magnetic fields and have no confining foils. Luginsland [2] have developed simple one-dimensional (1D) non-linear circuit equations that are solved numerically to estimate key device characteristics. This paper extends this approach to analytically estimate the values of the free parameters used in the circuit equations, compares the analytical values to similar values derived from two-dimensional (2D) particle-in-cell simulations, and compares the results of numerical solutions of the 1D circuit equations, 2D simulations, and initial experimental data. It is shown that the non-linear relationship between voltage and current emission in a space charge limited diode drives an RF oscillation whose frequency is determined by the resonant characteristics of the annular diode cavity. The results from the 1D analysis and 2D particle-in-cell (PIC) simulation are shown to be in excellent agreement.

I. BACKGROUND

RF oscillators using radially propagating planar electron beams were initially proposed by Varian [3] in 1941, as part of the work undertaken at Stanford University during development of the klystron. Not recognized at the time were many of the advantages inherent in such a radial geometry:

- Low electron source (diode) impedance due to the cylindrical geometry
- Simplified coupling between the RF generating device and the output structure (cylindrical device to cylindrical waveguide or transmission line)
- Minimal or no external magnetic field required to control electron beam instabilities

Other than the initial theoretical work by Varian, RF generation using radially propagating planar electron beams remained unexplored until the early 1990's. At that time, as an outgrowth to work simulating a cylindrically symmetric millimeter-wave transit time oscillator, Arman [4] noted the advantages of a radially propagating planar beam and developed a proposal for a radial klystron oscillator [5] independent of the earlier work by Varian. The radial klystron oscillator was further refined into the radial acceletron: a cylindrically symmetric transit time oscillator propagating a planar electron beam radially in the anode-cathode (A-K) gap. The development work by Arman was done entirely numerically, using the $2^{1}/_{2}$D (2D geometry, 3D fields) particle-in-cell (PIC) electromagnetic simulation code MAGIC [6].

II. 1-D CIRCUIT ANALYSIS

By modeling the radial acceletron as a nonlinear diode in series with an R-L-C circuit representing the resonant cavity, Luginsland derives the following small-signal equation governing the growth of RF oscillations within the cavity:

$$\left\{\frac{d^2}{dt^2}+\frac{\omega_0}{Q}\left[1-\frac{3}{2}\frac{R}{Z_D}\right]\frac{d}{dt}+\omega_0{}^2\right\}V_{rf}=0 \qquad \{1\}$$

For this derivation, ω_0 is the usual resonant frequency, Q is the cavity quality factor (including losses due to both wall conductivity and RF radiation), and Z_D is the impedance of the nonlinear diode. It is clear that the initial growth rate of RF oscillation is governed by the coefficient term associated with the first derivative:

$$1-\frac{3}{2}\frac{R}{Z_D}\langle 0 \Rightarrow V_{RF} \quad \text{growth} \qquad \{2\}$$

The RF voltage growth rate (e-folding time) is readily calculated to be:

$$\tau_{RF}=\frac{-2Q}{\omega_0\left[1-\frac{3}{2}\frac{R}{Z_D}\right]} \qquad \{3\}$$

Assuming the usual relationship between current and voltage in a space-charge-limited diode [7] and substituting into (2) gives an expression for the minimum current required for the onset of RF oscillation (note that in (4) the diode area has been collapsed into the perveance factor, 'P'):

$$I_{diode}=PV^{3/2} \Rightarrow Z_D=\frac{1}{P^{2/3}I^{1/3}} \qquad \{4\}$$

$$I\rangle\frac{8}{27}\frac{1}{R^3P^2} \quad \text{for RF growth} \qquad \{5\}$$

Note that to this point we have made no reference to the details of the currents and fields within the space-charge-limited diode; the onset of RF oscillation and subsequent growth is due entirely to the nonlinear nature of the diode and the diode interaction with the resonant cavity.

Inserting nominal values from the prototype radial acceletron design (Q=150, R=13.5 Ω, P=1.02e-4 in SI units, ω_0=2π*3.1e+9 sec^{-1}) will give estimates for the

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14. ABSTRACT
The transit-time effect in a coaxial structure has been used by Arman [l] to design low impedance high power microwave devices that use no externally generated magnetic fields and have no confining foils. Luginsland [2] have developed simple one-dimensional (1D) nonlinear circuit equations that are solved numerically to estimate key device characteristics. This paper extends this approach to analytically estimate the values of the free parameters used in the circuit equations, compares the analytical values to similar values derived from twodimensional (2D) particle-in-cell simulations, and compares the results of numerical solutions of the 1D circuit equations, 2D simulations, and initial experimental data. It is shown that the non-linear relationship between voltage and current emission in a space charge limited diode drives an RF oscillation whose frequency is determined by the resonant characteristics of the annular diode cavity. The results from the 1D analysis and 2D particle-in-cell (PIC) simulation are shown to be in excellent agreement.

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values of start current, required input voltage, and initial RF voltage growth rate:

$$I_{start} = 12 \text{ kA}$$
$$V = 235 \text{ kV}$$
$$Z_D = 20.2\Omega \text{ at } 235 \text{ kV}$$

Figures 1 and 2 plot the estimated start current and required voltage as a function of cavity Q; Figure 3 plots the estimated RF e-folding time as a function of driving voltage for several values of cavity Q. The above estimates are boundary values only; to obtain observable RF, the e-folding time for RF growth must be short enough for RF to grow from noise present in the diode. Requiring three e-folding times in the experimentally available 220 ns pulse width gives boundary values of 21 kA and 350 kV for observable RF.

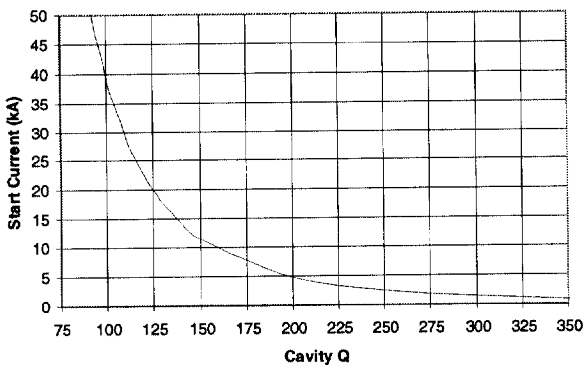

Figure 1. Estimated radial acceletron start current.

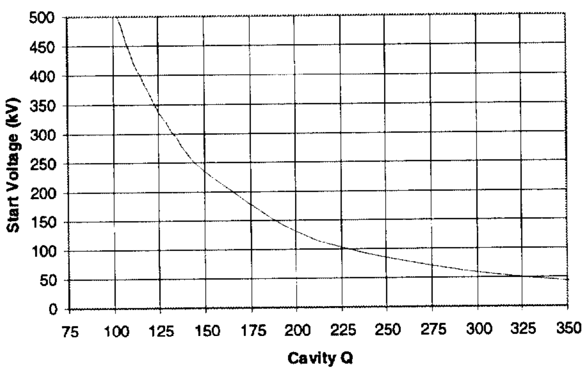

Figure 2. Estimated radial acceletron minimum voltage.

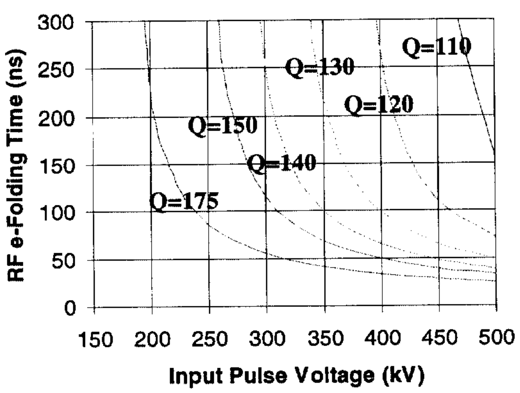

Figure 3. Radial acceletron estimated RF e-folding time.

III. CIRCUIT-MODEL PARAMETERS

The 1-D circuit equations used to model RF growth in the radial acceletron contain a number of free parameters: Q, R, Z_D, and ω_0. The parameters Q, R, and Z_D can be readily estimated from the design characteristics of device; a conceptual drawing of the prototype radial acceletron is shown in Figure 4, with a detail drawing of the actual prototype device in Figure 5. The resonant frequency, ω_0, is more problematic; the device requirements are that radial waveguide (lossy cavity) used to couple the A-K gap to the output transmission line must have a maximum in radial electric field at the A-K gap, and that the transit time of a space-charge limited electron across the A-K gap must be between one-half and one times the period of oscillation: π/ω_0 < gap crossing time < $2\pi/\omega_0$. The large physical size of the prototype compared to the wavelength of the desired microwave radiation results in a number of operating modes that satisfy these constraints; in the end, the value of ω_0 used in the circuit analysis remains an essentially free parameter.

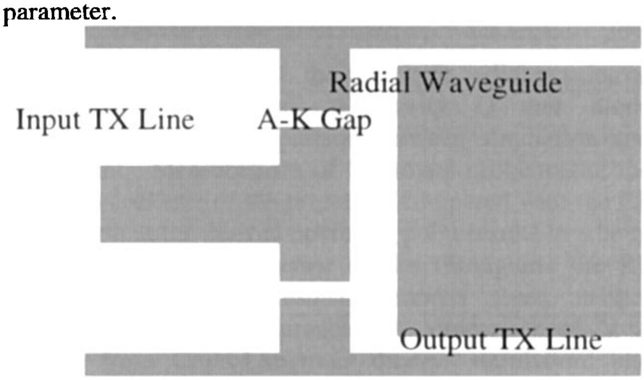

Figure 4. Radial acceletron conceptual drawing.

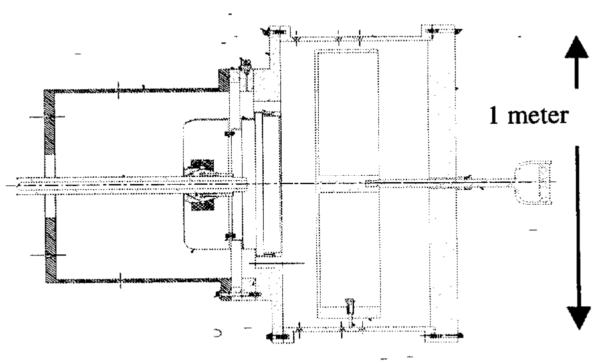

Figure 5. Prototype radial acceletron.

Using the Child-Langmuir expression for current in a space-charge-limited diode:

$$Z_D = \frac{1}{PV^{1/2}} \quad \{6\}$$

$$P = \frac{2.33 \times 10^{-6} S}{[\beta x]^2} \quad \text{(SI units)} \quad \{7\}$$

Here, S is the diode area, x is the A-K gap separation distance, and β is a constant equal to 1.0 for a planar gap, and calculated using the method of Langmuir for a gap consisting of two concentric cylinders (as in the radial acceletron).

The circuit parameters R, L, and C are related to Q by the usual relationship for a resonant parallel circuit: $Q = \omega_0 RC = R/\omega_0 L$. L and C are related by the usual expression for resonant frequency: $\omega_0^2 = {}^1/_{LC}$.

The capacitance C can be calculated directly given the geometry of the device, and R can be calculated by neglecting the contribution due to the resistivity of the metal wall, and treating the coupled cylindrical cavity and output transmission line as a radial waveguide coupled to a cylindrical transmission line. This coupled transmission line treatment results in an expression for R of the form:

$$R = \frac{Z_T}{[1-\Gamma]^2} \qquad \{8\}$$

In this expression, Z_T is the impedance of the output transmission line and Γ is the voltage reflection coefficient for the coupled radial waveguide-cylindrical transmission line.

A 1-D circuit-model analysis of the radial acceletron then can be accomplished by 1) choosing an appropriate value of ω_0 based on device geometry and/or simulation; 2) calculating values for R, C, and P based on device geometry; and, 3) calculating values for Q and Z_D for the desired input pulse voltage or current. For the prototype radial acceletron at 250kV input voltage, this results in $Q=110$; $R=18\ \Omega$, $Z_D=19.5\ \Omega$; these estimated values compare quite well with those derived from PIC simulation: $Q_{PIC}=150$, $R_{PIC}=13.5\ \Omega$, $Z_{D,PIC}=17.5\ \Omega$. At 350 kV input voltage, the analytic and PIC values of Q and R are unchanged, and the diode impedance becomes $Z_D=16.6\ \Omega$; $Z_{D,PIC}=16.0\ \Omega$.

IV. 2-D FINITE DIFFERENCE TIME DOMAIN SIMULATION

Simulation of the as-constructed prototype radial acceletron was done using MAGIC [6], a finite-difference, time-domain particle-in-cell electromagnetic simulation code. The software can be run either as a $2^1/_2$-D (2-D geometry, 3-D fields) simulation or as a fully 3-D simulation; to keep run times within reason, all simulations were done using the $2^1/_2$-D mode. Simulation runs were accomplished through the full scale of input pulse voltages achievable from the pulse power driver used by the experimental prototype: 150 kV through 500 kV. Simulations were done for two prototype device configurations: one with the extractor plate removed (thus eliminating the radial waveguide/output transmission line structure), and one with the extractor plate installed (the normal operating configuration). This was done to allow direct comparison between simulation and experimental measurements of the diode impedance (which were accomplished with the extractor plate removed).

With the extractor plate removed, the expected simulation result was a simple current rise to the space-charge-limited value, followed by a fall-off to zero at the end of the voltage pulse. Except for the low voltage simulations, this is what occurred. At input pulse voltages between 140 kV and 165 kV with the extractor plate removed, simulation showed an unexpected RF oscillation at 5.28 GHz. At this voltage level, the particle transit time is 2.7 times the period of RF oscillation—well out of the regime for transit-time amplification of RF fields. Particle plots showed no radial electron bunching, but strong oscillations in the space charge electron cloud along the device axis.

With the extractor plate installed, simulation showed 1) no RF output between 150 kV and 220 kV; 2) a slowly growing RF oscillation at 5.5–7.0 GHz for input pulse voltages between 230 kV and 330 kV (the RF frequency jumps around depending on input voltage); and, 3) the desired RF oscillation at approximately 3.15 GHz for input pulse voltages of 340 kV and greater.

V. COMPARISON OF EXPERIMENT AND SIMULATION

Initial experiments with the prototype radial acceletron emphasized measurement of cavity Q and diode impedance Z_D, and comparison between simulation and experiment. Measurement of Q proved problematic; the large physical size of the prototype compared with the RF wavelength at the desired operating point results in a large number of nearly degenerate modes throughout the RF spectrum. Individual resonant modes were neither distinguishable nor measureable. A comparison of Z_D as derived from Child-Langmuir theory, simulation, and experimental data is shown in Figure 6. The value of the 'P' parameter (related to diode perveance) is shown in Figure 7. An extensive experimental effort to identify the simulation-predicted 5.28 GHz RF was unsuccessful.

VI. ANALYSIS

From Figures 1 and 2, it can be readily seen that the required start current for RF growth, and the associated

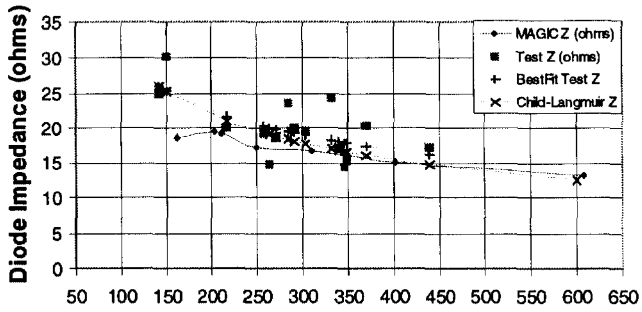

Figure 6. Radial acceletron diode impedance.

input pulse voltage, increase quite rapidly as the cavity Q decreases below approximately 175. The RF oscillation in the device builds from noise initially present in the A-K gap voltage and current; from Figure 3, the e-folding time associated with the RF growth increases significantly as the Q decreases. With an available current pulse length of ~220 ns (constrained by the pulse power blumlein section) and a maximum useable voltage of approximately 400 kV (constrained by arcing in the blumlein-radial acceletron transition section), it is clear that in order to obtain measureable RF output a minimum cavity Q of ~140 is required. From Figure 6, it is apparent that the MAGIC simulation consistently underestimates the diode impedance (or alternatively, overestimates the diode current) by approximately 10%; from equations (2) and (3), this underestimation of diode impedance will result in a corresponding underestimation of start current and overestimation of RF growth. With a simulation input pulse voltage of approximately 340 kV required for observable RF output, experimentally measurable RF will not be expected until input pulse voltages exceed approximately 380 kV. Note that the MAGIC simulations result in a non-zero slope for diode perveance as a function of diode voltage (Figure 7). The MAGIC simulation data is much better fit by a current-voltage relationship of $I=PV^{1.29}$, as contrasted with the classic Child-Langmuir exponent of 1.5. The reason for this difference is not yet understood.

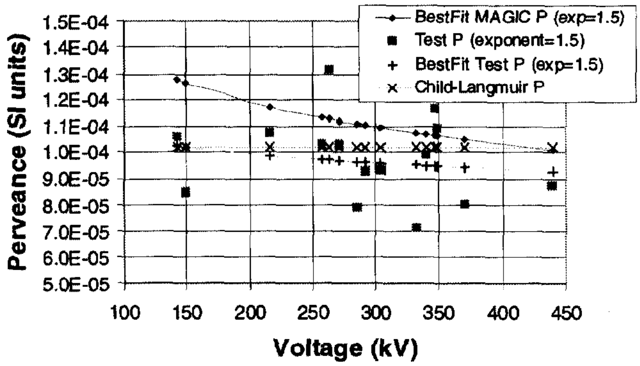

Figure 7. Radial acceletron diode perveance.

VII. CONCLUSIONS

We have extended the original 1-D circuit-model analyses of the radial acceletron to allow estimation of the values of the free parameters given design information from the as-constructed prototype device. The calculated values of the key parameters are shown to agree well with those derived from PIC simulation. Comparison of the results of simulation and actual experiment indicate that the simulation code consistently overestimates the diode current by approximately 10%; this overestimation of diode current will translate into a corresponding increase in start current for RF generation during actual experiments. With the current prototype design, the simulation prediction of observable 3.15GHz RF beginning at pulse voltages of 340 kV will likely translate into an experimental requirement for 370–380 kV pulse voltages—near the practical limit of the experimental set-up. The voltage requirement can be reduced by increasing the cavity Q, which requires adjusting the impedance mismatch between the radial waveguide and the output transmission line in the radial acceletron extractor section.

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